# *(Chapter 12)(Atoms)*

# XII

## Exercises

Question 12.1:

Choose the correct alternative from the clues given at the end of the each statement:

(a) The size of the atom in Thomson's model is .......... the atomic size in Rutherford's model. (much greater than/no different from/much less than.)

(b) In the ground state of .......... electrons are in stable equilibrium, while in .......... electrons always experience a net force.

(Thomson's model/ Rutherford's model.)

(c) A classical atom based on .......... is doomed to collapse.

(Thomson's model/ Rutherford's model.)

(d) An atom has a nearly continuous mass distribution in a .......... but has a highly nonuniform mass distribution in ..........

(Thomson's model/ Rutherford's model.)

(e) The positively charged part of the atom possesses most of the mass in ..........

(Rutherford's model/both the models.)

Answer

(a) The sizes of the atoms taken in Thomson's model and Rutherford's model have the same order of magnitude.

(b) In the ground state of Thomson's model, the electrons are in stable equilibrium. However, in Rutherford's model, the electrons always experience a net force.

(c) A classical atom based on Rutherford's model is doomed to collapse.

(d) An atom has a nearly continuous mass distribution in Thomson's model, but has a highly non-uniform mass distribution in Rutherford's model.

(e) The positively charged part of the atom possesses most of the mass in both the models.

Question 12.2:

Suppose you are given a chance to repeat the alpha-particle scattering experiment using a thin sheet of solid hydrogen in place of the gold foil. (Hydrogen is a solid at temperatures below 14 K.) What results do you expect?

Answer

In the alpha-particle scattering experiment, if a thin sheet of solid hydrogen is used in place of a gold foil, then the scattering angle would not be large enough. This is because the mass of hydrogen ($1.67 \times 10^{-27}$ kg) is less than the mass of incident α−particles ($6.64 \times 10^{-27}$ kg). Thus, the mass of the scattering particle is more than the target nucleus (hydrogen). As a result, the α−particles would not bounce back if solid hydrogen is used in the α-particle scattering experiment.

Question 12.3:

What is the shortest wavelength present in the Paschen series of spectral lines?

Answer

Rydberg's formula is given as:

$$\frac{hc}{\lambda} = 21.76 \times 10^{-19} \left[ \frac{1}{n_1^2} - \frac{1}{n_2^2} \right]$$

Where,

h = Planck's constant = $6.6 \times 10^{-34}$ Js

c = Speed of light = $3 \times 10^8$ m/s

($n_1$ and $n_2$ are integers)

The shortest wavelength present in the Paschen series of the spectral lines is given for values $n_1 = 3$ and $n_2 = \infty$.

$$\frac{hc}{\lambda} = 21.76 \times 10^{-19} \left[ \frac{1}{(3)^2} - \frac{1}{(\infty)^2} \right]$$

$$\lambda = \frac{6.6 \times 10^{-34} \times 3 \times 10^8 \times 9}{21.76 \times 10^{-19}}$$

$$= 8.189 \times 10^{-7} \text{ m}$$

$$= 818.9 \text{ nm}$$

Question 12.4:

A difference of 2.3 eV separates two energy levels in an atom. What is the frequency of radiation emitted when the atom makes a transition from the upper level to the lower level?

Answer

Separation of two energy levels in an atom,

E = 2.3 eV

$= 2.3 \times 1.6 \times 10^{-19}$

$= 3.68 \times 10^{-19}$ J

Let ν be the frequency of radiation emitted when the atom transits from the upper level to the lower level.

We have the relation for energy as:

E = hν

Where,

h = Planck's constant $= 6.62 \times 10^{-34}$ Js

$$\therefore \nu = \frac{E}{h}$$

$$= \frac{3.68 \times 10^{-19}}{6.62 \times 10^{-32}} = 5.55 \times 10^{14} \text{ Hz}$$

Hence, the frequency of the radiation is $5.6 \times 10^{14}$ Hz.

Question 12.5:

The ground state energy of hydrogen atom is −13.6 eV. What are the kinetic and potential energies of the electron in this state?

Answer

Ground state energy of hydrogen atom, E = − 13.6 eV

This is the total energy of a hydrogen atom. Kinetic energy is equal to the negative of the total energy.

Kinetic energy = − E = − (− 13.6) = 13.6 eV

Potential energy is equal to the negative of two times of kinetic energy.

Potential energy = − 2 × (13.6) = − 27 .2 eV

Question 12.6:

A hydrogen atom initially in the ground level absorbs a photon, which excites it to the n = 4 level. Determine the wavelength and frequency of the photon.

Answer

For ground level, $n_1 = 1$

Let $E_1$ be the energy of this level. It is known that $E_1$ is related with $n_1$ as:

$$E_1 = \frac{-13.6}{n_1^2}\text{ eV}$$

$$= \frac{-13.6}{1^2} = -13.6\text{ eV}$$

The atom is excited to a higher level, $n_2 = 4$.

Let $E_2$ be the energy of this level.

$$\therefore E_2 = \frac{-13.6}{n_2^2}\text{ eV}$$

$$= \frac{-13.6}{4^2} = -\frac{13.6}{16}\text{ eV}$$

The amount of energy absorbed by the photon is given as:

$E = E_2 - E_1$

$$= \frac{-13.6}{16} - \left(-\frac{13.6}{1}\right)$$

$$= \frac{13.6\times15}{16}\text{ eV}$$

$$= \frac{13.6\times15}{16}\times1.6\times10^{-19} = 2.04\times10^{-18}\text{ J}$$

For a photon of wavelength$\lambda$, the expression of energy is written as:

$$E = \frac{hc}{\lambda}$$

Where,

h = Planck's constant = $6.6 \times 10^{-34}$ Js

c = Speed of light = $3 \times 10^{8}$ m/s

$$\therefore \lambda = \frac{hc}{E}$$
$$= \frac{6.6\times10^{-34}\times3\times10^{8}}{2.04\times10^{-18}}$$
$$= 9.7\times10^{-8}\text{ m} = 97\text{ nm}$$

And, frequency of a photon is given by the relation,

$$\nu = \frac{c}{\lambda}$$
$$= \frac{3\times10^{8}}{9.7\times10^{-8}} \approx 3.1\times10^{15}\text{ Hz}$$

Hence, the wavelength of the photon is 97 nm while the frequency is $3.1 \times 10^{15}$ Hz.

Question 12.7:

(a) Using the Bohr's model calculate the speed of the electron in a hydrogen atom in the n = 1, 2, and 3 levels. (b) Calculate the orbital period in each of these levels.

Answer

(a) Let $v_1$ be the orbital speed of the electron in a hydrogen atom in the ground state level, $n_1 = 1$. For charge (e) of an electron, $v_1$ is given by the relation,

$$v_1 = \frac{e^2}{n_1 4\pi \epsilon_0 \left(\frac{h}{2\pi}\right)} = \frac{e^2}{2\epsilon_0 h}$$

Where, $e = 1.6 \times 10^{-19}$ C

$\epsilon_0$ = Permittivity of free space = $8.85 \times 10^{-12}\ N^{-1}\ C^2\ m^{-2}$

h = Planck's constant = $6.62 \times 10^{-34}$ Js

$$\therefore v_1 = \frac{\left(1.6\times10^{-19}\right)^2}{2\times8.85\times10^{-12}\times6.62\times10^{-34}}$$
$$= 0.0218\times10^{8} = 2.18\times10^{6}\ m/s$$

For level $n_2 = 2$, we can write the relation for the corresponding orbital speed as:

$$v_2 = \frac{e^2}{n_2 2 \in_0 h}$$
$$= \frac{\left(1.6\times10^{-19}\right)^2}{2\times2\times8.85\times10^{-12}\times6.62\times10^{-34}}$$
$$= 1.09 \times 10^6 \ m/s$$

And, for $n_3 = 3$, we can write the relation for the corresponding orbital speed as:

$$v_3 = \frac{e^2}{n_3 2 \in_0 h}$$
$$= \frac{\left(1.6\times10^{-19}\right)^2}{3\times2\times8.85\times10^{-12}\times6.62\times10^{-34}}$$
$$= 7.27 \times 10^5 \ m/s$$

Hence, the speed of the electron in a hydrogen atom in n = 1, n=2, and n=3 is $2.18 \times 10^6$ m/s, $1.09 \times 10^6$ m/s, $7.27 \times 10^5$ m/s respectively.

(b) Let $T_1$ be the orbital period of the electron when it is in level $n_1 = 1$.

Orbital period is related to orbital speed as:

$$T_1 = \frac{2\pi r_1}{v_1}$$

Where, $r_1$ = Radius of

the orbit

$$= \frac{n_1{}^2 h^2 \in_0}{\pi m e^2}$$

h = Planck's constant = $6.62 \times 10^{-34}$ Js
e = Charge on an electron = $1.6 \times 10^{-19}$ C
$\in_0$ = Permittivity of free space = $8.85 \times 10^{-12}$ $N^{-1}$ $C^2$ $m^{-2}$
m = Mass of an electron = $9.1 \times 10^{-31}$ kg

$$\therefore T_1 = \frac{2\pi r_1}{v_1}$$
$$= \frac{2\pi\times(1)^2\times\left(6.62\times10^{-34}\right)^2\times8.85\times10^{-12}}{2.18\times10^6\times\pi\times9.1\times10^{-31}\times\left(1.6\times10^{-19}\right)^2}$$
$$= 15.27\times10^{-17} = 1.527\times10^{-16} \ s$$

For level $n_2 = 2$, we can write the period as:

$$T_2 = \frac{2\pi r_2}{v_2}$$

Where, $r_2$ = Radius of the electron in $n_2 = 2$

$$= \frac{(n_2)^2 h^2 \epsilon_0}{\pi m e^2}$$

$$\therefore T_2 = \frac{2\pi r_2}{v_2}$$

$$= \frac{2\pi \times (2)^2 \times (6.62\times10^{-34})^2 \times 8.85\times10^{-12}}{1.09 \times 10^6 \times \pi \times 9.1\times10^{-31} \times (1.6\times10^{-19})^2}$$

$$= 1.22\times10^{-15}\ s$$

And, for level $n_3 = 3$, we can write the period as:

$$T_3 = \frac{2\pi r_3}{v_3}$$

Where,

$r_3$ = Radius of the electron in $n_3 = 3$

$$= \frac{(n_3)^2 h^2 \epsilon_0}{\pi m e^2}$$

$$\therefore T_3 = \frac{2\pi r_3}{v_3}$$

$$= \frac{2\pi \times (3)^2 \times (6.62\times10^{-34})^2 \times 8.85\times10^{-12}}{7.27 \times 10^5 \times \pi \times 9.1\times10^{-31} \times (1.6\times10^{-19})^2}$$

$$= 4.12 \times 10^{-15}\ s$$

Hence, the orbital period in each of these levels is $1.52 \times 10^{-16}$ s, $1.22 \times 10^{-15}$ s, and $4.12 \times 10^{-15}$ s respectively.

Question 12.8:

The radius of the innermost electron orbit of a hydrogen atom is $5.3 \times 10^{-11}$ m. What are the radii of the n = 2 and n =3 orbits?

Answer

The radius of the innermost orbit of a hydrogen atom, $r_1 = 5.3 \times 10^{-11}$ m.

Let $r_2$ be the radius of the orbit at n = 2. It is related to the radius of the innermost orbit as:

$$r_2 = (n)^2 r_1$$
$$= 4 \times 5.3 \times 10^{-11} = 2.12 \times 10^{-10}\ m$$

For n = 3, we can write the corresponding electron radius as:

$$r_3 = (n)^2 r_1$$
$$= 9 \times 5.3 \times 10^{-11} = 4.77 \times 10^{-10}\ m$$

Hence, the radii of an electron for n = 2 and n = 3 orbits are $2.12 \times 10^{-10}$ m and $4.77 \times 10^{-10}$ m respectively.

Question 12.9:

A 12.5 eV electron beam is used to bombard gaseous hydrogen at room temperature. What series of wavelengths will be emitted?

Answer

It is given that the energy of the electron beam used to bombard gaseous hydrogen at room temperature is 12.5 eV. Also, the energy of the gaseous hydrogen in its ground state at room temperature is −13.6 eV.

When gaseous hydrogen is bombarded with an electron beam, the energy of the gaseous hydrogen becomes −13.6 + 12.5 eV i.e., −1.1 eV.

Orbital energy is related to orbit level (n) as:

$$E = \frac{-13.6}{(n)^2}\ \text{eV}$$

For n = 3, $E = \frac{-13.6}{9} = -1.5\ \text{eV}$

This energy is approximately equal to the energy of gaseous hydrogen. It can be concluded that the electron has jumped from n = 1 to n = 3 level.
During its de-excitation, the electrons can jump from n = 3 to n = 1 directly, which forms a line of the Lyman series of the hydrogen spectrum.
We have the relation for wave number for Lyman series as:

$$\frac{1}{\lambda} = R_y\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$$

Where,
$R_y$ = Rydberg constant = $1.097 \times 10^7\ m^{-1}$ λ= Wavelength of radiation emitted by the transition of the electron For n = 3, we can obtain λ as:

$$\frac{1}{\lambda} = 1.097\times10^7\left(\frac{1}{1^2} - \frac{1}{3^2}\right)$$

$$= 1.097\times10^7\left(1 - \frac{1}{9}\right) = 1.097\times10^7\times\frac{8}{9}$$

$$\lambda = \frac{9}{8\times1.097\times10^7} = 102.55\ nm$$

If the electron jumps from n = 2 to n = 1, then the wavelength of the radiation is given as:

$$\frac{1}{\lambda} = 1.097\times10^7\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$$

$$= 1.097\times10^7\left(1 - \frac{1}{4}\right) = 1.097\times10^7\times\frac{3}{4}$$

$$\lambda = \frac{4}{1.097\times10^7\times3} = 121.54\ nm$$

If the transition takes place from n = 3 to n = 2, then the wavelength of the radiation is given as:

$$\frac{1}{\lambda}=1.097\times10^{7}\left(\frac{1}{2^2}-\frac{1}{3^2}\right)$$

$$=1.097\times10^{7}\left(\frac{1}{4}-\frac{1}{9}\right)=1.097\times10^{7}\times\frac{5}{36}$$

$$\lambda=\frac{36}{5\times1.097\times10^{7}}=656.33\ nm$$

This radiation corresponds to the Balmer series of the hydrogen spectrum.

Hence, in Lyman series, two wavelengths i.e., 102.5 nm and 121.5 nm are emitted. And in the Balmer series, one wavelength i.e., 656.33 nm is emitted.

Question 12.10:

In accordance with the Bohr's model, find the quantum number that characterises the earth's revolution around the sun in an orbit of radius $1.5 \times 10^{11}$ m with orbital speed $3 \times 10^4$ m/s. (Mass of earth = $6.0 \times 10^{24}$ kg.)

Answer

Radius of the orbit of the Earth around the Sun, r = $1.5 \times 10^{11}$ m

Orbital speed of the Earth, v = $3 \times 10^4$ m/s

Mass of the Earth, m = $6.0 \times 10^{24}$ kg

According to Bohr's model, angular momentum is quantized and given as:

$$mvr=\frac{nh}{2\pi}$$

Where,

h = Planck's constant = $6.62 \times 10^{-34}$ Js
n = Quantum number

$$\therefore n=\frac{mvr2\pi}{h}$$

$$=\frac{2\pi\times6\times10^{24}\times3\times10^{4}\times1.5\times10^{11}}{6.62\times10^{-34}}$$

$$=25.61\times10^{73}=2.6\times10^{74}$$

Hence, the quanta number that characterizes the Earth' revolution is $2.6 \times 10^{74}$.